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Mechanistic Classical Laboratory Situation with a Quantum Logic Structure

Diederik Aerts[1] and Bruno Van Bogaert[1]

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The difference between quantum entities and classical entities can be noticed in many different ways. Quantum logic has been profoundly interested in analyzing this difference and trying to understand it. Our aim is to represent a macroscopic classical mechanistic laboratory situation and to show that this situation entails a nonclassical logical structure. The example was presented some time ago by one of us, showing it to have a quantum probability model, and analyzing the effect of this on a possible understanding of the origin of quantum probabilities. In this paper we make a similar attempt, but now concentrate on the logical aspects of the example.

1. CLASSICAL ENTITIES AND QUANTUM ENTITIES

In the discipline of quantum logic the basic structure of research was originally the structure of the orthomodular lattice. Quantumlike entities had such a structure of an orthomodular lattice for the set of their propositions, while classical entities had a structure of a distributive orthomodular lattice (or Boolean algebra) for the set of their propositions. Meanwhile the classification of classical entities versus quantumlike (or nonclassical) entities has been studied in much more detail, relating the corresponding structures to real physical situations (e.g., Foulis and Randall, 1978, 1981; Aerts, 1981, 1983; Piron, 1976, 1990). We have taken a very easy criterion from the results of this research on the possibility of distinguishing between the two kinds of entities, classical and nonclassical.

We shall consider an entity (classical or nonclassical) to be described by a set **M** (denoted by $m, n, \ldots$) of measurements and a set Σ (denoted by $p, q, \ldots$) of states. In different approaches different names have been given to these two basis sets (measurements have been called yes–no experiments,

[1]Department of Theoretical Physics, Vrije Universiteit Brussel, 1050, Brussels.

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questions, propositions, observables, and operations, and states have also been called preparations), but mainly these differences will play no role in what we would like to show in this paper. The results can easily be translated in the proper approach. We must remark that when we use the concept "state," we mean "pure state." The more general situation of mixed states will not be considered here, since our example in any case does not contain mixed states. The following characterization of a classical entity shall be adopted [it is the characterization explicitly used in Aerts (1981, 1983)]:

Criterion for classical character of an entity. An entity, described by a set of measurements **M** and a set of states Σ, is a classical entity if for any state p (pure state), for an arbitrary measurement m, the outcome of the measurement can be predicted with certainty (probability equal to 1).

This definition is certainly satisfied for all the classical entities described by classical mechanics, since the measurements are then represented by real-valued functions on the phase space of states, and indeed to a certain state p there corresponds one point of phase space, and every measurement (observable) has one value, which is the outcome that can be predicted with certainty. For quantum entities described by quantum mechanics this definition is obviously not satisfied. Indeed, for a state represented by a state vector that is not an eigenstate of a certain measurement, represented by a self-adjoint operator, no prediction of any outcome can be given.

Moreover, we will explicitly construct the lattice of propositions going together with the entity of the example and see that it is a pure quantum logic lattice (distributivity is not satisfied).

2. PRESENTATION OF THE EXAMPLE

The classical macroscopic spin model that we will present in this section has been presented in Aerts (1986, 1987, 1991) with the aim of giving a possible explanation for the nonclassical character of the quantum probabilities. It is shown in Aerts (1986, 1987, 1991) that a lack of knowledge about the measurements on a physical entity gives rise to a collection of probabilities connected to this entity that is nonclassical. It is also shown that the nonclassical probability calculus of quantum mechanics can be interpreted as being the result of a lack of knowledge about the measurements. In this paper we describe in much more detail the measurement apparatus, the preparation apparatus, the measurements, and the preparations, such that it is shown that the relative frequencies of repeated experiments of this classical situation indeed lead to the same collection of probabilities as the one of the spin of a spin-1/2 quantum entity [this detailed description can also be found in Aerts (1991)]. And then we will see that also the criterion for a classical entity on the level of quantum logic is not satisfied for this example, such that we really can speak of a macroscopic mechanistic quantum logic example.

We first give a detailed description of the construction of the measurement apparatus $MA(\mathbf{a})$. We have a rigid rod constituted of a nonconducting material (for example, some plastic) of a certain length l (see Figure 1). At the endpoints of the rod are two little iron balls, ball 1 and ball 2. In the laboratory where our experiment will be performed we have a battery at our disposal that contains a certain fixed amount of negative charge s. There is a mechanism that brings both of the iron balls during a certain time in contact with the battery, such that they get charged. As a result ball 1 is charged with a certain charge s_1 and ball 2 with a charge $s-s_1=s_2$. Then the balls get uncharged again. And immediately after, they get charged again, and then uncharged again. And so on. The rigid rod is placed fixed in the laboratory such that ball 1 is in space direction $\mathbf{a}$ and ball 2 in space direction $-\mathbf{a}$ in a plane orthogonal to some fixed direction $\mathbf{x}$. The physical entities that we consider are little iron balls positively charged with a fixed charge q. They can be put in the neighborhood of the measurement apparatus and connected to it by a nonconducting rigid rod of length $l/2$. The measurement $m(\mathbf{a})$ consists of letting go the positive charge q. It will be attracted by the two negative charges of the measuring apparatus by Coulomb forces F_1 and F_2. We suppose that this happens in a viscous medium, such that under the influence of friction, finally the positively charged ball q will end up at one

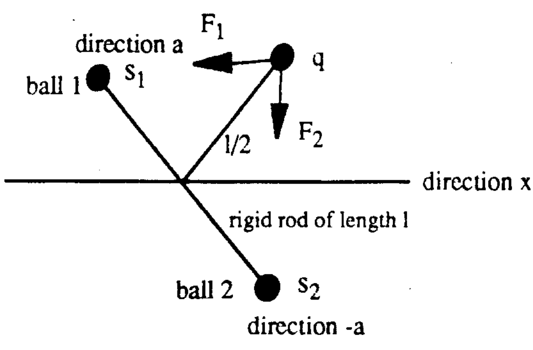

Fig. 1. The measuring apparatus $MA(a)$ consists of a rigid rod of length l with two iron balls, at the endpoints ball 1 and ball 2, alternatively charged and decharged by an amount of negative charge s, such that ball 1 gets one part s_1 of the charge and ball 2 the other part $s_2=s-s_1$. The measurement $m(a)$ consists of connecting a positively charged ball q at the center of the rigid rod of the measuring apparatus by means of a rigid rod of length $l/2$. The charge q will be captured by one of the two negative charges. If it is captured by s_1, then we give outcome "a-up" to measurement $m(a)$; if it is captured by s_2, then we give outcome "a-down" to the measurement $m(a)$.

of the balls of the measuring apparatus. If it ends up at ball 1, we give the outcome "**a**-up" and if it ends up at ball 2 we give the outcome "**a**-down" for the measurement $m(\mathbf{a})$.

To make the analogy with the Stern–Gerlach measurements on the spin of a quantum entity complete, we suppose that we cannot directly manipulate the charge q (as we cannot directly manipulate a quantum entity's spin). Hence the preparation of the state of q has to be done by means of a preparation apparatus $PA(\mathbf{b})$. Exactly as for the spin case, the preparation apparatus is slightly different from the measurement apparatus. We set at work the part of the measuring apparatus $MA(\mathbf{b})$ which is the rigid rod with its two charged balls 1 and 2 on the ball q, but only conserve those balls q that have been captured by ball 1 of the measuring apparatus (the balls q that are captured by ball 2 are considered to be annihilated). Hence after the preparation with apparatus $PA(\mathbf{b})$, the charge q is at a location in direction b and length $l/2$ from the center of the measurement apparatus rod. We call this state $p(\mathbf{b})$ (see Figure 2).

An experiment $e(\mathbf{a}, \mathbf{b})$ consists of a preparation of state $p(\mathbf{b})$ followed by a measurement $m(\mathbf{a})$. We can now start making the repeated experiments. In relation with the state $p(\mathbf{b})$ and afterward the measurements $m(\mathbf{a})$, we can count the number $N(\mathbf{a}\text{-up})$ of outcomes "**a**-up" or the number $N(\mathbf{a}\text{-down})$ of outcomes "**a**-down" and divide by the total number N of classical entities that have participated in the repeated experiments. If the relative frequencies $\nu(\mathbf{a}\text{-up})$ and $\nu(\mathbf{a}\text{-down})$ approximate real numbers between 0 and 1 if N goes to infinity, then we call these real numbers the probabilities $P(\mathbf{a}\text{-up})$ and $P(\mathbf{a}\text{-down})$. We can introduce the following probabilities: $P(\mathbf{a}, \mathbf{b}) =$ the probability that if the classical entity C is prepared with the state $p(\mathbf{b})$, and the measurement $m(\mathbf{a})$ is performed, the outcome "**a**-up" will occur and hence the probability that the experiment $e(\mathbf{a}, \mathbf{b})$ yields the outcome "**a**-up"; $P(-\mathbf{a}, \mathbf{b}) =$ the probability that if the classical entity C is prepared with the state $p(\mathbf{b})$, and the measurement $m(\mathbf{a})$ is performed, the outcome "**a**-down" will occur and hence the probability that the experiment $e(\mathbf{a}, \mathbf{b})$ yields the outcome "**a**-down." To determine these probabilities, we must go to a laboratory and perform the repeated experiments, and then see what we find for

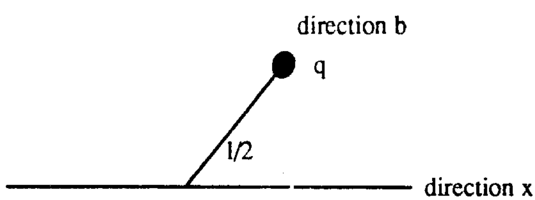

Fig. 2. The preparation (state) $p(b)$ by means of the preparation apparatus $PA(b)$.

the relative frequencies. If we do this, we find that they will depend on the angle between the two space directions **a** and **b**, and if we take $P(\mathbf{a}, \mathbf{b}) = \cos^2(\gamma/2)$, where γ is the angle between the space directions **a** and **b**, we are in good agreement with the experimental findings.

We have constructed an experimental situation using only mechanistic classical macroscopic entities that is similar to the experimental situation of the spin measurement by a Stern–Gerlach apparatus on a quantum entity of spin 1/2. Both lead to the same set of probabilities. The probabilities of the quantum example can be derived also theoretically, using the quantum mechanical description of the Stern–Gerlach measurement situation. But it must be pointed out that this quantum calculation does not give any specification on the physical mechanism by which the quantum measuring apparatus (the Stern–Gerlach apparatus) affects the quantum entity to lead to one of the possible outcomes. This mechanism, taking into account the enormous complexity of the Stern–Gerlach apparatus, is probably very complicated, and full of hidden randomness. For our classical example we know the mechanism, and hence can propose a classical model for it. Of course, like every theoretical model (also the quantum model), this model is an idealization of the reality happening in the laboratory. It can be interpreted as a kind of simple model for the working of the quantum Stern–Gerlach apparatus. Let us regard the measurement situation of our classical macroscopic example a little bit closer, and see what model we can propose. The three charges are located in a plane, the positive charge q, prepared by $p(\mathbf{b})$, at a point indicated by the direction **b**, and the two negative charges of $m(\mathbf{a})$ at diametrically opposed points indicated by the directions **a** and $-\mathbf{a}$ (see Figure 3). Let us call γ the angle between the two space directions **a** and **b**. The forces F_1 and F_2 are the coulomb forces; hence

$$|F_1| = \frac{s_1 \cdot q}{\pi \varepsilon_0 l^2 \sin^2(\gamma/2)}, \qquad |F_2| = \frac{s_2 \cdot q}{\pi \varepsilon_0 l^2 \cos^2(\gamma/2)} \tag{1}$$

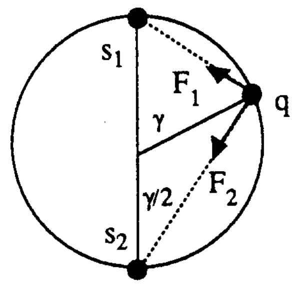

Fig. 3. We consider the three charges from Figure 1 as they are located in a plane; q is the classical entity located at the point indicated by the direction b, s_1 and s_2 of the measuring apparatus are located at the points indicated by the directions a and $-a$. F_1 and F_2 are the forces of attraction between s_1 and s, and s_2 and s.

The charge q will move under the influence of the two fixed charges s_1 and s_2 of the measurement apparatus, and finally will arrive at rest at one of the two places where s_1 or s_2 is located. We propose the following model:

1. We suppose that the place where finally the charge q will end up is determined by the magnitude of the forces of attraction between the three charges. Namely, if $|F_1|$ is bigger than $|F_2|$, q will move and arrive at the place where s_1 is located, and if $|F_1|$ is smaller than $|F_2|$, q will move and arrive at the place where s_2 is located.

2. The charges s_1 and s_2 are in a certain sense arbitrary. Because at the moment where the charge q is put in the measurement apparatus, we do not know in which state the charging and uncharging of the two charged balls, 1 and 2, of the measuring apparatus is. If this happens at a moment of unchanged charges s_1 and s_2, nothing will happen with the charge q (since there are no forces). The action will only start after the next charging. In any case, since we wait until one of the two outcomes is registered, this experimental situation can be modeled by supposing that for the actual motion of the charge q, and hence for the occurrence of one of the outcomes, s_1 and s_2 are such that s_1 is a random number in the interval $[0, s]$, and $s_2 = s - s_1$.

By means of hypotheses 1 and 2, we can give a mathematical derivation for the probabilities $P(\mathbf{a}, \mathbf{b})$ and we see that we find indeed the already experimentally approximated probabilities:

$$\begin{aligned}
P(\mathbf{a}, \mathbf{b}) &= \text{Probability that } |F_1| \text{ is bigger than } |F_2| \\
&= P\left(\frac{s_1 \cdot q}{\pi \varepsilon_0 l^2 \sin^2(\gamma/2)} > \frac{s_2 \cdot q}{\pi \varepsilon_0 l^2 \cos^2(\gamma/2)}\right) \\
&= P\left(s_1 \cos^2\left(\frac{\gamma}{2}\right) > s_2 \sin^2\left(\frac{\gamma}{2}\right)\right) \\
&= P\left(s_1 \cos^2\left(\frac{\gamma}{2}\right) > (s - s_1) \sin^2\left(\frac{\gamma}{2}\right)\right) \\
&= P\left(s_1 > s \sin^2\left(\frac{\gamma}{2}\right)\right) \\
&= \frac{s - s \cdot \sin^2(\gamma/2)}{s} = \cos^2\left(\frac{\gamma}{2}\right) \qquad (2)
\end{aligned}$$

3. THE QUANTUM LOGICAL STRUCTURE OF THE EXAMPLE

Because these experimental probabilities $P(\mathbf{a}, \mathbf{b})$, through the relative frequencies of repeated measurements, are the **only** connection of a theory with the experiments in our reality, we can describe this classical entity by the spin formalism of quantum mechanics. Concretely this means that a state of the iron ball of positive charge q in the direction **b** can be represented by a unit vector u_b of a two-dimensional complex Hilbert space. If $\mathbf{b}=(\cos\phi\cdot\sin\theta, \sin\phi\cdot\cos\theta, \cos\theta)$ where (θ, ϕ) are the spherical coordinates in some fixed coordinate system with origin at the center of the rigid rod of the measuring apparatus, then we can represent the state of the classical entity by the unit vector

$$u_b=(e^{-i\phi/2}\cdot\cos(\theta/2), e^{i\phi/2}\cdot\sin(\theta/2)) \tag{3}$$

of a two-dimensional complex Hilbert space as is also done for the spin state of the spin of a spin-1/2 quantum entity. And the measurement $m(\mathbf{a})$ is represented by means of the self-adjoint operator

$$S_{\alpha\beta}=\begin{pmatrix} \cos\alpha & e^{-i\beta}\sin\alpha \\ e^{i\beta}\sin\alpha & -\cos\alpha \end{pmatrix} \tag{4}$$

where $\mathbf{a}=(\cos\beta\cdot\sin\alpha, \sin\beta\cdot\sin\alpha, \cos\alpha)$. The eigenvalue $+1$ corresponds to the outcome "**a**-up" and the eigenvalue -1 to the outcome "**a**-down" of the measurement $m(\mathbf{a})$. The ordinary rules of the quantum mechanical calculations lead to the corresponding probabilities found as limits of the relative frequencies of the repeated measurements $m(\mathbf{a})$ if the entity is prepared with preparation $p(\mathbf{b})$. Also, all the other happenings that can be imagined for the case of the spin of the quantum entity of spin 1/2 have their counterpart in our classical model. For example, performing the measurement $m(\mathbf{a})$ on the classical entity in state u_b prepared by the preparation $p(\mathbf{b})$ indeed changes this state u_b into another state, depending on the outcome. The state after the measurement is u_a if the outcome "**a**-up" has occurred, and is u_{-a} if the outcome "**a**-down" has occurred. This change of state, which "really" happens in our classical example, is what often is referred to as the "collapse of the wave function" in the quantum language. This change of state is governed in our classical example by the action of the Coulomb forces, and hence does not happen instantaneously. In the quantum mechanical situation this change of state should be governed by the interaction between the measurement apparatus $A(Q, \mathbf{a})$, Stern–Gerlach magnet + screen, with the quantum mechanical entity. We do not know the nature of this interaction, and hence cannot speculate about the "speed" with which it operates.

We have constructed a situation with a macroscopic classical entity that can be described by the quantum formalism in a two-dimensional complex Hilbert space. The lattice of propositions that goes together with this macroscopic example is then obviously also the same as the lattice of propositions about the spin of a quantum entity of spin 1/2, which, as is well known, is a pure quantum logic lattice (distributivity is not satisfied).

On the other hand, the criterion for the classical character of an entity that we mentioned in Section 1 is obviously not satisfied in the example. Indeed if the entity is in a state (pure state) $p(\mathbf{b})$ for a measurement $m(\mathbf{a})$ with $\mathbf{a}$ different from $\mathbf{b}$ and $-\mathbf{b}$, we cannot predict the outcome. Both outcomes are possible with the probabilities that we have mentioned. The nonclassical aspect is due to the presence of a hidden variable in the measuring apparatus (or to the presence of hidden measurements). We would like now to analyze the plausibility of the presence of this nonclassical aspect of "hidden measurements" for an eventual explanation of the quantum logical structure of quantum entities in general.

4. THE PLAUSIBILITY OF OUR EXPLANATION

The explanation for the quantum structure that we propose is only a *possible* explanation. We cannot prove that it is the correct one, and we certainly do not pretend it to be "complete." But we should like to end this section by indicating that our explanation as to the quantum structure of the probabilities, the non-Boolean structure of the lattice of propositions, and the nonvalidity of the criterion for the classical character of an entity is very plausible from a general physical point of view. We repeat: We explain the presence of these quantum structures by the fact that there is a lack of knowledge on the state of the measuring apparatus, or on the interaction between the measuring apparatus and the physical entity during the measurement. Let us show that this explanation is very plausible: If we introduce the probability as approximate relative frequencies of repeated equivalent experiments, we have to be aware that the technique that we use in real laboratory circumstances to decide that we indeed are performing a series of repeated experiments that are "equivalent" experiments depends in a very complicated and conventional way [in the sense used by Poincaré (1902) and in the sense analyzed in detail in Aerts (1992)] on our prior knowledge of those pieces of reality that we use to construct the experiment, and which we accept to be equivalent. In the example that we have given, we have subdivided an experiment into two parts, a preparation of the physical entity and a measurement. Again for these two parts we can make the same remark. The technique that we use to decide that the preparations and measurements of repeated experiments are equivalent depends in a very complicated and conventional way on our prior knowledge of those pieces of reality used to construct the preparations and the measurements.

The case where preparations (states) are not really equivalent on a deeper level has been treated in general by classical probability theory. In this case we say in classical physics that the physical entity is prepared in a "mixed" state and not a "pure" state. The pure states describe the deeper underlying reality for the mixed state. The formalization of this situation leads to a classical statistical theory, making use of classical Kolmogorovian probability models, and giving rise to a classical logical structure. The case where the measurements are not really equivalent on a deeper level has not been treated systematically, and cannot be treated by a classical probability theory once we want to consider several measurements that cannot be executed together. This is exactly the situation that we have artificially created in our macroscopic classical example, and we see that this situation can be described by the quantum mechanical formalism. Indeed we consider repeated measurements $m(\mathbf{a})$ as equivalent *only* for the fact that the direction of space determined by the rod that connects the two negative charges is the same. But as we have constructed the measurement apparatus we can see that on a deeper level, these measurements are not the same. The way in which the charge s will be distributed on the two balls 1 and 2 is in principle different for every measurement. It is this nonequivalence of the measurement on a deeper level that is at the origin of the presence of the probabilities that we can describe by the quantum mechanical formalism. We could say, $m(\mathbf{a})$ is a "mixed" measurement. And the pure measurements would be the ones where we control the distribution of the charge s over the two balls of the measurement apparatus.

Now the last question, is this explanation plausible for the situation of a quantum entity? To answer this question, let us regard the situation of the Stern–Gerlach measurement connected with the spin. Indeed, also in this situation, we consider repeated measurements as equivalent *only* because the direction $\mathbf{a}$ of the constant part of the magnetic field used in the Stern–Gerlach magnet + screen is the same. We do not know anything about the reality of this Stern–Gerlach magnet + screen on deeper levels. Since the Stern–Gerlach magnet + screen is a construction made of an enormous amount of atoms, it is "sure" that all these atoms will never be found in the same state for a repeated measurement. Hence it is very plausible that there are deeper "significant" levels, and that this Stern–Gerlach measurement is also a "mixed" measurement. If our explanation for the origin of the presence of the quantum probabilities is correct, the situation in a quantum measurement would be so that we imagine always making the same measurement in the repeated experiments, but on a deeper level this is not true. And this fact gives rise to the quantumlike probabilities, and to quantum logical structures, exactly as in the case of the explicit macroscopic classical example. The measuring apparatus $MA(\mathbf{a})$ that we have introduced for our classical example can even be adapted as a phenomenological model for the Stern–Gerlach + screen.

This manner of the interpreting probabilities, and quantum logic, would also explain why this probability structure, and this quantum logic structure, is only found for the description of the entities in the microscopic world. Indeed, in this microscopic world, we make experiments with macroscopic entities, and since the measurement apparatus is macroscopic while the physical entity is microscopic, very often the lack of a deeper-level description of the measuring apparatus introduces these quantumlike probabilities. In our macroscopic example we have imitated such a situation. If we accept our explanation, we can conclude by stating that the general structure of a probability model, and the general structure of a logic, connected with a general physical situation should be quantumlike. The classical probability structures, and the classical logics, represent the special situations where we can manipulate the measurement apparatus in such a detailed way that almost really equivalent measurements can be performed in the repeated experiments, and probability is only introduced from the presence of mixed states. These ideas are explored in a more complete way in Aerts (1992).

REFERENCES

Aerts, D. (1981). The one and the many. Towards a unification of the quantum and classical description of one and many physical entities, Doctoral dissertation, Vrije Universiteit Brussel.

Aerts, D. (1983). Classical theories and non-classical theories as a special case of a more general theory, *Journal of Mathematical Physics*, **24**, 2441.

Aerts, D. (1986). A possible explanation for the probabilities of quantum mechanics, *Journal of Mathematical Physics*, **27**, 203.

Aerts, D. (1987). The origin of the non-classical character of the quantum probability model, in *Information Complexity and Control in Quantum Physics*, A. Blanquiere, S. Diner, and G. Lochak, eds., Springer-Verlag.

Aerts, D. (1991). A macroscopic classical laboratory situation with only macroscopical classical entities giving rise to a quantum mechanical probability model, in *Quantum Probability*, Volume VI, Luigi Accardi, ed., World Scientific.

Aerts, D. (1992). The construction of reality and its influence on the understanding of quantum structures, *International Journal of Theoretical Physics*, this issue.

Foulis, D. J., and Randall, C. H. (1978). Manuals morphism, and quantum mechanics, in *Mathematical Foundations of Quantum Theory*, A. Marlow, ed., Academic Press, New York.

Foulis, D. J., and Randall, C. H. (1981). What are quantum logics and what ought they to be? in *Current Issues in Quantum Logic*, E. Beltrametti and B. C. van Fraassen, eds., Plenum Press, New York.

Piron, C. (1976). *Foundations of Quantum Physics*, Benjamin, Reading, Massachusetts.

Piron, C. (1990). *Mécanique Quantique, Bases et Applications*, Press Polytechnique et Universitaire Romande, Lausanne.

Poincaré, H. (1902). *La science et l'hypothèse*, Flammarion, Paris.